Department of Computer Science and Information Systems
CSIS 6924 (Algorithms),
CSIS 0324 (Topics in Theoretical Computer Science)
Assignment 1

Deadline: October 21, 1:30pm.
We accept both printed and written answers.
Remember to put your name and university number in the answer script.
Submit it to assignment box A6 (lift lobby of 3/F, Chow Yei Ching building).

Answer all questions. They carry the same marks.
For CSIS 0324 students, please note the additional questions overleaf.

1. [p333, Ex 17.1-2] Suppose that we have a set of activities to schedule among a large number of lecture halls. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture halls. Prove that the algorithm is optimal.

2. [p333, Ex 17.1-3] Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give an example to show that the approach of selecting the activity of least duration from those that are compatible with previously selected activities does not work. Do the same for the approach of always selecting the activity that overlaps the fewest other remaining activities.

3. [p350, Ex 17.4-2, modified] Let S denotes a set of real-valued non-zero row vectors. A subset S_0 of S is said to be in $\mathcal{I}$ if no vector in S_0 is a linear combination of the other vectors in S_0. By using this definition (and no other theorems of linear algebra), show that $(S, \mathcal{I})$ is a matroid.

4. [p350, Ex 17.4-5] Show how to transform the weight function of a weighted matroid problem, where the desired optimal solution is a *minimum-weight* maximal independent subset, to make it an standard weighted-matroid problem. Argue carefully that your transformation is correct. (Note: a maximal independent subset is an independent subset S_0, such that if we add any element in $S - S_0$ to S_0, it becomes dependent.)

5. [p354, Prob 17-1] (Coin changing) Consider the problem of making change for n cents using the least number of coins.

   (a) Describe a greedy algorithm to make change consisting of quarters (25 cents), dimes (10 cents), nickels (5 cents) and pennies (1 cent). Prove that your algorithm yields an optimal solution.

   (b) Suppose that the available coins are in the denominations c^0, c^1, $\ldots$, c^k for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.

   (c) Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution.

Additional questions for CSIS 0324 students

1. [p350, Prob 17.4-4] Let S be a finite set and let S_1, S_2, $\ldots$, S_k be a partition of S into nonempty disjoint subsets. Define the structure $(S, \mathcal{I})$ by the condition that $\mathcal{I} = \{A : |A \cap S_i| \leq 1$ for $i = 1, 2, \ldots, k\}$. Show that $(S, \mathcal{I})$ is a matroid. That is, the set of all sets A that contain at most one member in each block of the partition determines the independent sets of a matroid.

2. [Due to p354, Prob 17-2] The first parts of problem 17-2 in the text looks like this:

   > Let $G = (V, E)$ be an undirected graph, and let $\mathcal{I}$ be the set of acyclic subsets of E. The ***incidence matrix*** for G is a $|V| \times |E|$ matrix M such that $M_{ve} = 1$ if edge e is incident on vertex v, and $M_{ve} = 0$ otherwise. Argue that a set of columns of M is linearly independent if and only if the corresponding set of edges is acyclic. Then, use the result of Exercise 17.4-2 (question 3) to provide an alternate proof that $(E, \mathcal{I})$ is a matroid.

   Show that the part is wrong, by showing that there is some set of columns that are linearly independent, but yet they form a cycle in the graph.

   Show that changing the definition of M from real-valued matrix to boolean matrix, with the following table of addition and multiplication, fixes the problem.

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

   That is, provide a definition of linear independence on boolean matrices, show that such linear independence actually forms a matroid for boolean matrices, and show that the independence of a set of edges in G is equivalent to the independence of a set of columns in a boolean matrix.